A review study on the bare hull form equations of submarine

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Abstract
This paper presents some of the equations of the bare hull form of the submarines. Design of overall hydrodynamic shape, is an important start point of submarine design process, specially in hydrodynamics and general arrangement. This paper has collected the most famous of these equations that are common practice in real naval submarines and ROVs. In naval architecture design, there are some main criteria for selecting an optimum hull form for a submarine, that is hinted in this paper. This paper is a review study.

Keywords: Submarine, Hull, Form, Shape, equation.

Introduction
There are some rules and concepts about submarines and submersibles shape design. There is urgent need for understanding the basis and concepts of shape design. Submarine shape design is strictly depended on the hydrodynamics such as other marine vehicles and ships. Submarines are encountered to limited energy in submerged navigation and because of that, the minimum resistance is vital in submarine hydrodynamic design. In addition, the shape design is depended on the internal architecture and general arrangements of submarine. Convergence between hydrodynamic needs and architecture needs are vital for determination of overall shape design of submarine. Submarine have two modes of navigation: surfaced mode and submerged mode. In surfaced mode of navigation, the energy source limitation is lesser than submerged mode. Therefore, in real naval submarines, the base of determination of the hull form, is submerged mode. The several parts of submarine are bare hull and sailing. The parts of bare hull are the bow, middle part and stern. In some forms, there is not middle body part, thus bare hull is direct connection of bow to the stern. The focus of this paper is on this type of bare hull.

Refs [1,2] are the main references that describe the notes of naval submarine shape design with regarding the hydrodynamic aspects. In Refs [3], there are the basis of submarine shape selection with all aspects such as general arrangement, hydrodynamic, dynamic stability, flow noise and sonar efficiency. Ref.[4] contains a lot of scientific materials about naval submarine hull form and appendages design with hydrodynamic considerations. Some studies about submarine hull form design with minimum resistance by CFD method is done in Ref [5-10] by M.Moonesun and colleagues. Special discussions about naval submarine shape design are presented in Iranian Hydrodynamic Series of Submarines (IHSS)[6,11].

In Ref[12,13] some case study discussions about the hydrodynamic effects of the bow shape and overall length of the submarine by CFD method are presented. Defence R&D Canada, suggested a hull form equation for bare hull, sailing and appendages [14,15] as the name of "DREA standard model". Refs.[16-18] presents an equation for teardrop hull form with the limitations of their coefficients but the main source of their equation is presented in Ref.[19], and the simulation of the hull form with different coefficients is presented in Ref.[20]. Other equation for torpedo hull shape is presented in Ref[21]. Formula "Myring" as a famous formula for axisymmetric shapes is presented in Ref.[22]. Extensive experimental results about hydrodynamic optimization of teardrop or similar shapes are presented in Ref.[23] as a main reference book in the field of the selection of aerodynamic and hydrodynamic shapes based on experimental tests. A collective experimental study about the shape design of bow and stern of the underwater vehicles are presented in Ref [24] that are based on the underwater missiles but the most parts of this book, is practicable in naval submarine shape design. Another experimental
studies on the several teardrop shapes of submarines are presented in Ref.[25]. In Refs[26,27], all equations of hull form, sailing and appendages are presented with experimental and CFD result for SUBOFF project.

**Some important factors in bare hull form design**
Bare hull, is an outer hydrodynamic shape that envelopes the pressure hull. For a well judgment and the best selection of bare hull form, the most important factors in bare hull form design are counting as: 1) minimum submerged resistance: the ratio L/D and bow shape are the important factors. Demand for minimum resistance in submerged navigation is versus surfaced navigation but in submarine resistance calculation, the main criterion is submerged mode. Optimization of submarine shape, based on minimum resistance is represented in Refs.[28,29] with a logical algorithm. Optimization of shape based on minimum resistance in snorkel depth is shown in Ref.[30]. Optimization of shape in surface condition (such as ships) is not regarded because in new modern submarines with using high storage batteries or nuclear storage or fuel cells, there isn't any need to surfacing, and air suction is done by snorkel mast in snorkel depth. 2) general arrangement demands specially for D. 3) enough volume for providing enough buoyancy according to given weight. 4) minimum flow noise specially around sonar and acoustic sensors. 5) minimum cavitation around propeller. 6) suitable for single hull or twin hull: in single hull submarine, there is almost cylindrical pressure hull, and hydrodynamic envelope there is only in the bow and stern parts. In twin hull submarine, hydrodynamic envelope (light hull), envelopes the pressure hull, totally. The shape demands of these two kinds of hull are different.

**Bare hull form equations**
As mentioned in "Introduction", there are several sources about equations of bare hull form which will be presented here.

A) According to Refs.[14,15]: The equations are presented as "DREA Model" that is shown in Fig.1 and includes the specification of bare hull and appendages.

The DREA model is specified in three sections; bow, midbody and tail. The fineness is L/D=8.75 so that bow length is equal to 1.75D and midbody length is 4D and stern length is 3D. Axisymmetric profile of bow is:

$$ \frac{r}{D} = 0.8685 \sqrt{\frac{x}{L}} - 0.3978 \frac{x}{D} + 0.006511 \left( \frac{x}{D} \right)^2 + 0.005086 \left( \frac{x}{D} \right)^3 \quad (1) $$

Middle body is a cylinder and axisymmetric parabolic profile of stern is:

$$ \frac{r}{D} = \frac{1}{3} \left( \frac{x}{L} \right) - \frac{1}{18} \left( \frac{x}{D} \right)^2 \quad (2) $$

All parameters are shown in Fig.1 [14]. These equations are rewrited to another face in Ref.[15] as a function of length but by the same dimension relations (L/D=8.75). For bow with the length equal to 0.2L (0<x/L<0.2), the equation is:

$$ \frac{r_1(x)}{L} = \frac{D}{L} \left[ 2.56905 \sqrt{\frac{x}{L}} - 3.4805 \frac{x}{L} + 0.49848 \left( \frac{x}{L} \right)^2 + 3.40732 \left( \frac{x}{L} \right)^3 \right] \quad (3) $$

Coordinate is shown in Fig.2. Middle body part, is a cylinder part and equation is (0.2<x/L<1-3D/L):

$$ \frac{r_2(x)}{L} = \frac{D}{2L} \quad (4) $$

The stern part with the length equal to 3D, the equation is (1-3D/L <x/L<1):
\[
\frac{r_2(x)}{L} = \frac{D}{2L} \left( \frac{x}{L} - \left( 1 - \frac{3D}{L} \right)^2 \right)
\]  
(5)

Alternate stern profile by using \(x' = L - x\) and \(0 < x/D < 3\):
\[
\frac{r_3(x')}{D} = \frac{1}{2} - \frac{1}{18} \left( 3 - \frac{x'}{D} \right)^2
\]  
(6)

Appendages are specified as: all appendages have four digit NACA foils, which hydrofoil thickness profile is given by:
\[
\frac{Y_t}{c} = \pm \frac{t}{c} \sqrt{1.4845 \frac{R}{c} - 0.63 \frac{X_f}{L_f} - 1.758 \left( \frac{X_f}{L_f} \right)^2 + 1.4215 \left( \frac{X_f}{L_f} \right)^3 - 0.5075 \left( \frac{X_f}{L_f} \right)^4}
\]  
(7)

Where the "c" is local chord length and \(t/c\) is the maximum thickness to chord ratio. The leading edge is at \(x = 0\) and the trailing edge, which has non zero thickness is at \(x = c\). Tail planes are four identical rudder and stern plane appendages in a symmetrical "+" configuration (Fig.3). The sections are flat tip NACA0015 thickness profile (\(t/c=0.15\)). Propeller hub, is at aft three percent of hull. For sail, there is rectangular planform, flat tip, NACA0020 thickness profile (\(t/c=0.2\)). Sail planes (Fig.4) are flat tips, NACA0015 thickness profile (\(t/c=0.15\)).

B) According to Refs.[16-20]: The equations are presented as "Hull Envelope Equation". The envelope is first developed as a pure tear drop shape with the forward body comprising 40 percent of the length and the after body comprising the remaining 60 percent [18]. The forward body is formed by revolving an ellipse about its major axis and is described by the following equation:
\[
Y_f = R \left[ 1 - \left( \frac{X_f}{L_f} \right)^{n_f} \right]^{1/n_f}
\]  
(8)

The after body is formed by revolving a line around axis and is described by:
\[
Y_a = R \left[ 1 - \left( \frac{X_a}{L_a} \right)^{n_a} \right]
\]  
(9)

The quantities \(Y_a\) and \(Y_f\) are the local radius of the respective body of revolution with \(X_a\) and \(X_f\) describing the local position of the radius along the body (Fig.5). If parallel middle body is added to the envelope, then cylindrical section with a radius equal to the maximum radius of the fore and after body is inserted in between them. The local radii represent the offsets for drawing the submarine hull and also determine the prismatic coefficient for the hull section. The prismatic coefficient (\(C_p\)) is a hull form parameter for fullness and is the ratio of volume of the body of revolution divided by the
volume of a right cylinder with the same maximum radius. For an optimum shape, the fore and after bodies will have different values for \( C_p \). \( C_p \) is used to determine the total hull volume by the following relation:

\[
\text{Volume} = \frac{\pi D^2}{4} \left[ 3.6D_C_{pa} + \left( \frac{L_D}{D} - 6 \right) + 2.4D_C_{pf} \right]
\]  

(10)

Where the added term \( (L/D-6)D \) accounts for the volume of the parallel middle body where \( C_p=1 \). The surface area for the body can be described by the following relation:

\[
\text{Wetted Surface} = \pi D^2 \left[ 3.6D_C_{sa} + \left( \frac{L_D}{D} - 6 \right) + 2.4D_C_{sf} \right]
\]  

(11)

Surface coefficient (\( C_S \)), describes the ratio of the surface area of the body to the surface area of a cylinder with the same maximum radius. The factors \( n_f \) and \( n_a \) in equations, describe the “fullness” of the body by affecting the curvature of the parabolas. Tables 1 lists some representative values for \( n_f \) and \( n_a \) along with their resultant \( C_p \) and \( C_s \). Figure 6 illustrates the effect of varying \( n_f \) and \( n_a \) on the hull geometry [18].

Table 1: Selected values for \( C_p \) and \( C_s \)

<table>
<thead>
<tr>
<th>Fore body</th>
<th>After body</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_f )</td>
<td>2.0</td>
</tr>
<tr>
<td>( C_s )</td>
<td>6697</td>
</tr>
<tr>
<td>( C_s )</td>
<td>0.7999</td>
</tr>
</tbody>
</table>

The range of these parameters, regarded for sample, represented in Fig.7. The simulation of the hull form with different coefficients is presented in Fig.8. These equations are re writed to another face in Refs.[17,29,30] for another coordinate origin (Fig.9), and the shape optimization is done for snorkeling in snorkel depth.

\[
r_a = R \left( 1 - \left( \frac{L_a - x}{L_a} \right)^{n_a} \right)
\]  

(12)

\[
r_f = R \left( 1 - \left( \frac{x - L_a - L_f}{L_f} \right)^{n_f} \right)^{1/n_f}
\]  

(13)

C) According to Refs.[21,22]: The equations are presented as "Myring Equations" for earning minimum resistance and many submarines, AUVs and UUVs are designed according to these equations such as REMUS [21] which describes a body contour with minimal drag coefficient for a given fineness ratio (maximum length to maximum diameter). The parameters "a,b,c,d,θ" are shown in Fig.10. Parameter "n" is an exponential parameter which can be varied to give different body shapes. These equations assume an origin at the nose of the vehicle. Nose shape is given by the modified semi-elliptical radius distribution.
Tail shape is given by the equation:

\[ r(\Xi) = \frac{1}{2}d \left[ 1 - \left( \frac{\Xi + \alpha_{\text{offset}} - a}{a} \right)^2 \right] \]  

Where the forward body length is:

\[ l_f = a + b - \alpha_{\text{offset}} \]  

Table 2 gives the dimensionalized "Myring" parameters.

| A | \( n_1 = 1.84 \) |
|   | \( n_2 = 1.84 \) |

| B | \( n_1 = 2.00 \)  |
|   | \( n_2 = 2.00 \)  |

| C | \( n_1 = 2.50 \) |
|   | \( n_2 = 2.50 \) |

| D | \( n_1 = 3.00 \) |
|   | \( n_2 = 3.00 \) |

| E | \( n_1 = 4.00 \) |
|   | \( n_2 = 2.75 \) |

| F | \( n_1 = 2.50 \) |
|   | \( n_2 = 2.75 \) |

\( a \left[ \frac{3d}{2c^2} - \tan \theta \right] (\Xi - l)^2 + \left[ \frac{d}{c^3} - \frac{\tan \theta}{c^2} \right] (\Xi - l)^3 \)  

Table 2: Myring parameters for STD REMUS [21]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+1.91e-001</td>
<td>m</td>
<td>Nose Length</td>
</tr>
<tr>
<td>( \alpha_{\text{offset}} )</td>
<td>+1.56e-002</td>
<td>m</td>
<td>Nose Offset</td>
</tr>
<tr>
<td>( b )</td>
<td>+6.54e-001</td>
<td>m</td>
<td>Midbody Length</td>
</tr>
<tr>
<td>( c )</td>
<td>+8.41e-001</td>
<td>m</td>
<td>Tail Length</td>
</tr>
<tr>
<td>( \alpha_{\text{offset}} )</td>
<td>+3.86e-002</td>
<td>m</td>
<td>Tail Offset</td>
</tr>
<tr>
<td>( n )</td>
<td>+2.00</td>
<td>n/a</td>
<td>Exponential Coefficient</td>
</tr>
<tr>
<td>( \theta )</td>
<td>+4.35e-001</td>
<td>radians</td>
<td>Included Tail Angle</td>
</tr>
<tr>
<td>( d )</td>
<td>+9.1e-001</td>
<td>m</td>
<td>Maximum Hull Diameter</td>
</tr>
<tr>
<td>( l_f )</td>
<td>+8.28e-001</td>
<td>m</td>
<td>Vehicle Forward Length</td>
</tr>
<tr>
<td>( l )</td>
<td>+1.33e+000</td>
<td>m</td>
<td>Vehicle Total Length</td>
</tr>
</tbody>
</table>

**D) According to Refs.[26,27]:** The equations are presented as "SUBOFF Model" from Defence Advanced Research Project Agency (DARPA) that is shown in Fig.11 with coordinate location. Two geometrically identical models designed to a linear scale ratio of 24 with detailed equations and shape specifications for computer programming and modeling in CFD and experimental model test [26]. An extensive hydrodynamic results are presented in Ref.[27].

![SUBOFF hull and coordinate](image)

The two SUBOFF models differ only in the location of surface pressure taps. Each model component is described by equation giving either the axial and radial values for an axisymmetric component and all units in equation are in feet. The model has overall length of 14.291667 ft (4.356 m) and maximum diameter of 1.666667 ft (0.508 m). Dimensions of model and equations to define axisymmetric hull are:
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Forebody Length = 3.333333 ft (1.016 m)  
Parallel Middle Body Length = 7.3125 ft (2.229 m)  
Afterbody Length = 3.645833 ft (1.111 m)  
Aft Perpendicular at x = 13.979167 ft (4.461 m)  
Total Body Length = 14.291666 ft (4.356 m)  
Maximum Body Diameter = 1.666667 ft (0.508 m)  
\( \lambda = \text{(full/model) Scale Ratio} = 24 \)

Bow equation in \( 0<x<3.333 \) (ft) is:
\[
r = R \left\{ 1.126395101x(0.3x - 1) + 0.442874707x^2(0.3x - 1)^3 + 1 - (0.3x - 1)^4(1.2x + 1) \right\}^{1/2} 
\]

(17)

Parallel middle body is a cylinder in \( 3.333<x<10.645833 \) (ft). Stern equation in \( 10.645833<x<13.979167 \) (ft) is:
\[
r_s = R \left\{ r_h^2 + r_h K_\alpha \xi^2 \left( 20 - 20r_h^2 - 4r_h K_\alpha - \frac{1}{3} K_1 \right) \xi^3 + (-45 + 45r_h^2 + 6r_h K_\alpha + K_0) \xi^4 + (36 - 36r_h^2 - 4r_h K_\alpha - K_1) \xi^5 \right\}^{1/2} 
\]

(18)

\[
r_h = 0.1175 \quad K_\alpha = 10 \quad K_0 = 44.6244 \quad K_1 = 13.979167 - x, \text{ in feet} \]

(19)

The equation of stern cap in \( 13.979167<x<14.291667 \) (ft) is:
\[
r_{sc} = 0.1175 R \left\{ 1 - (3.2x - 44.733333)^2 \right\}^{1/2} 
\]

(20)

The profile of each part is as Fig.12.

![Figure 12: Hull form of SUBOFF [26]](image)

The sail is defined by 4 sections: fore body, parallel middle body, after body and cap (Fig.13). Its main dimensions are:

| Sail Forebody Length = 0.325521 ft (0.099m) |
| Sail Parallel Middle Body Length = 0.200521 ft (0.061m) |
| Sail Afterbody Length = 0.682292 ft (0.208m) |
| Total Sail Length = 1.208333 ft (0.368m) |
| Span of Sail with Uniform Profile = 0.674479 ft (0.206 m) |
| \( Z_{max} = \text{One-Half the Maximum Sail Thickness} = 0.109375 \) (0.033m)

Sail fore body equation is:
\[
Z_t = Z_{max} \left[ 2.094759(A) + 0.2071781(B) + (C) \right]^{1/2} 
\]

(21)

\[
A = 2D \quad (D - 1)^4 \quad B = \frac{1}{3} (D^2)(D - 1)^3 \quad C = 1 - (D - 1)^4(4D + 1) \quad D = 3.072.(x - 3.032986) 
\]
Sail parallel middle body equation is:

\[ Z_1 = Z_{\text{max}} = 0.109375 \text{ ft} = 1.3125 \text{ inch} \]  

(22)

Sail aft body equation is:

\[ Z_1 = 0.1093750 \left[ 2.238361 (E(E - 1)^4) + 3.106529 (E^2(E - 1)^3) + (1 - (E - 1)^4(4E + 1)) \right] \]  

(23)

\[ E = (4.241319 - x)/0.6822917 \]  

(24)

Sail is closed at top with an ellipsoid that names "Sail cap" and is defined as:

\[ Z_2 = \left( Z_1^2 - (2(y - 1.507813))^2 \right)^{1/2} \]  

(25)

\[ Z_1 \] was defined previously as a function of \( x \). The intersection of hull and sail is:

\[ [R_{HB}(x)]^2 = y^2 + Z_1^2 \]  

(26)

Figure 13: Sail form of SUBOFF [26]

**CFD Analysis**

For selecting a good shape of submarine bow, several factors should be regarded which mentioned before, but one of the main factors are "minimum resistance". Evaluation of resistance of different shapes of submarines by CFD method is performed in Ref.[5-10,31] by the authors of this paper. Here, for example, the results of simulation of bow shape, with different coefficients of "\( n_f \)" are represented but extensive discussions are presented in Ref.[31].

This analysis is done by Flow Vision (V.2.3) software based on CFD method and solving the RANS equations. Generally, the validity of the results of this software has been done by several experimental test cases, and nowadays this software is accepted as a practicable and reliable software in CFD activities. For modeling these cases in this paper, Finite Volume Method (FVM) is used. A structured mesh with cubic cell has been used to map the space around the submarine. For modeling the boundary layer near the solid surfaces, the selected cell near the object is tiny and very small compared to the other parts of domain.

For selecting the proper quantity of the cells, for one certain model (Model1) and \( v=10 \) m/s, six different amount of meshes were selected and the results were compared insofar as the results remained almost constant after 1.2 millions meshes, and it shows that the results are independent of meshing. In all modeling the mesh quantities are considered more than 2 millions. For the selection of suitable iteration, it was continued until the results were almost constant with variations less than one percent, which shows the convergence of the solution. All iterations are continued to more than one millions. In this domain, there is inlet (with uniform flow), Free outlet, Symmetry (in the four faces of the box) and Wall (for the body of submarine). Dimensions of cubic domain are 42m length (equal to 7L), 6m beam and 6m height (equal to L or 6D). Pay attention to that only quarter of the body is modeled because of axis-symmetry. Meanwhile, the study has shown that the beam and height equal to 6D in this study can be acceptable. Here, there are little meshes in far from the object. The forward distance of the model is equal to 3L and after distance is 3L in the total length of 7L (Fig.14). The turbulence model is K-Epsilon and \( y+ \) is considered equal to 30. The considered flow is incompressible fluid (fresh water) in 20 degrees centigrade and constant velocity of 10 m/s.

Figure 14: (a) Domain and structured grid (b) Very tiny cells near the wall for boundary layer modeling and keeping y+ about 30 (c) Quarterly modeling because of axis-symmetry
Results of modeling are represented in Tab.3 and Fig.15 for different values of \( n_f \) based on "Equation 8". These results show that for \( n_f=1.85 \), is the worst selection in the range of common values of \( n_f \) between 1.35~2. In Tab.3 the value \( C_v \) is resistance coefficient based on \( Vol^{2/3} \) instead of wetted area (\( A_w \)). Parameter V/C in this table, regarded the provided volume by the bow. In this modeling, the length is constant but volume varies. Total length is 6m and bow length is 3m. Complete specifications of modeling is presented in Ref.[31].

Table 3: Resistance and resistance coefficient of bow for different values of \( n_f \)

<table>
<thead>
<tr>
<th>( n_f )</th>
<th>( R_t )</th>
<th>( A_w )</th>
<th>volume</th>
<th>( C^*1000 )</th>
<th>( V/(C^*100) )</th>
<th>( C_v^*1000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1944</td>
<td>11.16</td>
<td>2.09</td>
<td>3.484</td>
<td>59.99</td>
<td>23.80</td>
</tr>
<tr>
<td>1.15</td>
<td>1820</td>
<td>11.79</td>
<td>2.26</td>
<td>3.087</td>
<td>73.20</td>
<td>21.15</td>
</tr>
<tr>
<td>1.35</td>
<td>1876</td>
<td>12.48</td>
<td>2.45</td>
<td>3.006</td>
<td>81.49</td>
<td>20.66</td>
</tr>
<tr>
<td>1.5</td>
<td>1952</td>
<td>12.9</td>
<td>2.58</td>
<td>3.026</td>
<td>85.25</td>
<td>20.77</td>
</tr>
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<td>1.65</td>
<td>2060</td>
<td>13.25</td>
<td>2.68</td>
<td>3.109</td>
<td>86.19</td>
<td>21.37</td>
</tr>
<tr>
<td>1.75</td>
<td>2200</td>
<td>13.45</td>
<td>2.75</td>
<td>3.271</td>
<td>84.06</td>
<td>22.43</td>
</tr>
<tr>
<td>1.85</td>
<td>2264</td>
<td>13.63</td>
<td>2.8</td>
<td>3.322</td>
<td>84.28</td>
<td>22.81</td>
</tr>
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<td>2196</td>
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<td>3.167</td>
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<td>93.38</td>
<td>22.58</td>
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<td>2724</td>
<td>14.87</td>
<td>3.21</td>
<td>3.664</td>
<td>87.62</td>
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</tr>
<tr>
<td>4</td>
<td>3180</td>
<td>15.36</td>
<td>3.37</td>
<td>4.141</td>
<td>81.39</td>
<td>28.32</td>
</tr>
<tr>
<td>5</td>
<td>3500</td>
<td>15.64</td>
<td>3.46</td>
<td>4.476</td>
<td>77.31</td>
<td>30.62</td>
</tr>
</tbody>
</table>

Figure 15: Resistance coefficient of the bow for the different values in the range of values of "\( n_f \)" between 1~3 (based on Equation 8)

**Conclusion**

In this paper, a review study on the equations of bare hull of submarines and their appendages and sailing was represented. These are the most famous equations in submarine form design. Bare hull, is an outer hydrodynamic shape that envelopes the pressure hull. For a well judgment and the best selection of the bare hull form, the most important factors in bare hull form design must be counted such as: minimum submerged resistance, general arrangement demands, providing enough buoyancy, minimum flow noise, minimum cavitation around propeller and suitable for single hull or twin hull submarine.

**Nomenclature**

- \( C_s \): Surface coefficient
- \( C_p \): Prismatic coefficient
- \( D \): Maximum diameter of the outer hull
- \( I_{HSS} \): Iranian Hydrodynamic Series of Submarines
- \( R \): Maximum radius of the outer hull
- \( x_a \): X from stern
- \( x_f \): X from bow
- \( Y_a \): Y from axis in bow
- \( Y_f \): Y from axis in stern
- \( L \): Overall length of hull
- \( L_a \): Length of aft (stern)
- \( L_f \): Length of forward part (bow)

**References**