Solving the hub-and-spoke problems with application in the marine transportation system

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Abstract

Networks involving hubs are important in transportation and telecommunications. In both settings, when there is traffic between several origins and several destinations, there are economic benefits if this traffic is concentrated through some nodes and/or arcs of the network. A hub (such as port in marine transportation) is a node of the network that concentrates traffic from several origins and distributes it to the final destinations. Hub-and-spoke problem is NP-hard problem that frequently applies in the design of transportation and distribution systems. This paper has focused on the Single Allocation Hub Location Problem (SAHLP). We have investigated some models with un-capacitated SAHLP that have been modeled by four indices. Many indices cause to get large model and to get sluggish model solving. Thus, we modeled Single Allocation Hub Location Problems with two indices in order to get fast model solving.

Keywords: Hub-and-spoke, Network design, Hub location problem, Un-capacitated.

Introduction

The hub and spoke distribution paradigm (or model or network) is a system of connections arranged like a chariot wheel, in which all traffic moves along spokes connected to the hub at the center. The model has commonly used in industry, in particular in transport, telecommunications and freight, as well as in distributed computing. For a network of n nodes, only (n – 1) routes are necessary to connect all nodes, that is, the upper bound is (n – 1), and the complexity is O (n). This compares favorably to the n(n-1)/2 routes, or O (n2), that would be required to connect each node to every other node in a point-to-point network [1]. Hub-and-spoke networks play an important role in many transportation systems. These networks provide efficient transportation between many origins and destinations via a set of hubs that serve as switching and flow consolidation points. Hub networks can provide service among many origins and destinations with fewer arcs (lines) than in a point-to-point network and thus can reduce transportation costs by exploiting the economies of scale from consolidated flows [2].

In 1929 United Parcel Service (UPS) became the first package delivery company to provide air service via privately operated airlines [1]. UPS's Parcel Network is based on a hub and spoke model. UPS operates centers which feed parcels to hubs where parcels are sorted and forwarded to their destinations. In the mid-1970s FedEx adopted the hub and spoke model for overnight package delivery, and after the airline industry was deregulated in 1978, hub and spoke paradigm was annexed by several airlines. Airlines have extended the hub and spoke model in various ways. One method is to create additional hubs on a regional basis, and to create major routes between the hubs. This reduces the need to travel long distances between nodes that are close together. Another method is to use focus cities to implement point-to-point service for high traffic routes, bypassing the hub entirely. Because of the rich applications in the real world, studies on various hub location models have attracted much attention since O'Kelly [2].

Hub Location Problems (HLP) are classical combinatorial optimization problems that arise in telecommunication and transportation networks where nodes send and receive commodities (i.e., data transmissions, passengers, express packages, mail, etc.) through special facilities or transshipment points called hubs. Hubs consolidate flows from origin nodes and re-route them to destination nodes sometimes via other hubs. The sending and receiving nodes in such networks are called spokes. The networks are called hub-spoke networks [3].

The assumption in hub-spoke networks is that, hubs are fully-connected through low-cost high-volume pathways that allow a discount factor to be applied to the transportation cost of the flow between a given hub pair. Another assumption in these networks is that, all the inter-nodal flow takes place through at least one hub and at most two [4]. Broadly, the hub location problem (HLP) is concerned with locating hubs on the network and allocating spokes to the hubs so as to minimize total flow cost subject to the above assumptions [5].

Hub-and-spoke networks have application in many areas. Common examples include passenger airlines or marine lines [5,6], express package delivery firms [7,8], supply-chain of chain stores such as Wal-mart [9,10], and many other areas. Many studies have indicated that the implementation of hub-and-spoke network has improved the performance of the distribution system. Hub location problem has many varieties according to the constraints and decision variables involved such as the way of selecting the number of hubs to be located, the way of spokes are assigned to hubs, the existence of capacity limits on hubs, etc. A comprehensive survey on Hub Location Problems (HLPs) and their...
classification can be found in Almur and Kara [11]. In the Single Allocation Hub Location Problem (SAHLP), a spoke has allocated to one hub exactly and the number of hubs to be used is not known in advance.

Furthermore, hubs are capacitated or un-capacitated. Capacitated hubs can handle limited inter-nodal flow whereas un-capacitated hubs can't manage any amount of flow. Corresponding to these hub types, two variants of SAHLP exist; the Capacitated Single Allocation Hub Location Problem (CSAHLP) with capacity limits on hubs; the Un-capacitated Single Allocation Hub Location Problem (USAHLP) involving hubs with unlimited capacities. An example of the capacitated SAHLP application is in postal delivery systems in which a sorting center (or hub) sorts and consolidates mail arriving from different postal districts and re-route it to the destination [12] usually through other centers. The sorting centers in such systems have capacities i.e., they can handle a maximum amount of mail flows from origin destination points. Example of the application of the un-capacitated SAHLP is the marine transportation networks [2]. Marine transportation network is a distribution center. In this network usually capacity of output is larger than amount of input. Therefore, we don’t consider capacity for these networks.

The SAHLP is NP-hard problem [13]. Additionally, the number of hubs is not known a priori in SAHLP and the single assignment constraint i.e., a given spoke must be allocated to only one hub, holds. Due to their usefulness and economic importance, both the capacitated and un-capacitated versions of SAHLP have received a good amount of research attention and exact and heuristic methods have been proposed to tackle them.

Many models and algorithms have been proposed for the hub location problem on a network. A good classification of the models can be found in Campbell [14] and [15], and a complete review of models and algorithms in Bryan and O’Kelly [16]. Kara and Tansel [17], study the p-hub center problem, which minimizes the maximum dissatisfaction of passengers in air travel (or maximum travel time). Most of these models can be written in single allocation versions (a demand node is allocated to one hub exactly) or multiple-allocation versions (a demand node is allocated to several hubs, depending on the destination of the traffic). With the exception of O’Kelly [18], who minimizes variability of hub usage, none of the hub models consider congestion. Some of the models, as Ernst and Krishnamoorthy [12] and Ebery et al. [19] include capacity limits, in terms of the traffic each hub receives from the nodes allocated to it.

The Single Allocation Hub Location Problem is NP-hard combinatorial optimization problem. Due to this complexity, solving with exact methods is computationally intractable especially when large problem instances are involved. Therefore, in recent years, meta-heuristics such as Genetic Algorithms [7,2], Tabu Search [20], and Ant Colony Optimization algorithms [21] have been proposed for the SAHLP. But different models have been proposed for small and medium problem instances.

Ernst et al. [12] proposed a mixed integer formulation for the CSAHLP and developed two heuristic algorithms for the problem based on simulated annealing and random descent. They used the upper bound obtained with SA-RDH to develop an LP-based branch and bound solution method for CSAHLP. Ernst also introduced the AP (Australian Post) benchmark data for the Capacitated Single Allocation Hub Location Problem (CSAHLP). Matsubayashi [22] studied a cost allocation problem arising from hub–spoke network systems and formulated this problem as a cooperative game and analyzed the core allocation, which was a widely used solution concept. Elhedhli and Hu [23] considered a hub-and-spoke network design problem with congestion. They proposed a model extending current models by taking congestion effects into account, which was achieved through a non-linear cost term in the objective function. Lin and Chen [24] proposed a generalized hub-and-spoke network in a capacitated and directed network configuration that integrated the operations of three common hub-and-spoke networks: pure, stopover and center directs.

Klincewica [13] described an algorithm, based on dual ascent and dual adjustment techniques within a branch-and-bound scheme, for the un-capacitated hub location problem. Mr. He [25] addressed a hub-and-spoke network problem for railroad freight, where a central planner was to find transport routes, frequency of service, length of trains and transportation quantity to be used.

For un-capacitated Single Allocation Hub Location Problems, Abdinour-Helm [10] proposed a hybrid approach based on GA and Tabu Search to solve. The GA was used to determine the number and location of hubs and the Tabu Search (TS), to assign spokes to hubs. They reported an improvement over their earlier GA-approach that used distance-based assignment of spokes to hubs. However, their stand-alone GA results are not available. Topcuoglo et al. [26] developed a GA-based approach to the USAHLP. They found improved solutions to some Civil Aviation Board (CAB) problems. They also used Australian Post (AP) data in their experiments that had not been previously used in any study on USAHLP. The non-GA heuristics applied to the USAHLP include two hybrid approaches by Chen et al. [29]. They combined SA with Tabu List (TL) to solve USAHLP. This approach involves applying Simulated Annealing to determine an upper-bound for the number of hubs and then using restricted single location exchange procedure to locate the hubs. Non-hub nodes are first allocated to nearest hubs followed by an improvement procedure for allocation that iteratively re-allocates nodes with less flow to other hubs until no improvement is possible.

The Single Allocation Hub Location Problem (SAHLP)

The single allocation hub location problem is a special type of hub location problem in which a spoke can be assigned to only a single hub. Moreover, the number of hubs is a decision variable in SAHLP and a fixed cost for establishing a hub is also included in the overall transportation cost. A single allocation hub-and-spoke network has shown in the Figure 1.
The objective in the SAHLP is to minimize the cost of establishing hubs and cost of transportation. This is subject to the constraints that a spoke must be assigned to only a single hub and hub capacities must not be exceeded.

The SAHLP has two varieties: (1) the un-capacitated Single Allocation Hub Location Problem (USAHLP) and (2) the capacitated Single Allocation Hub Location Problem (CSAHLP). It uses the SAHLP formulation for the Un-capacitated Single Allocation Hub location Problem proposed by Naeem [28]. USAHLP is a mixed integer formulation and can be found in Naeem's thesis. The formal description is given below:

\[
\begin{align*}
\min Z &= \sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} W_{ij} (x_{d_{ik}} + \beta_{d_{ij}}) X_{ijk} + \sum_{k=1}^{N} F_{k} Z_{kk} \quad (1) \\
\sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} W_{ij} X_{ijk} &= 1 \quad (2) \\
Z_{ik} &\leq Z_{kk} \quad (3) \\
\sum_{i=1}^{N} \sum_{j=1}^{N} (W_{ij} X_{ijk} + W_{ji} X_{jik}) &= \left( \sum_{i=1}^{N} W_{ij} + \sum_{j=1}^{N} W_{ji} \right) Z_{ik} \quad (4) \\
Z_{ik} &\in \{0,1\} \quad (5) \\
0 &\leq X_{ijk} \leq 1 \quad (6)
\end{align*}
\]

Where:
- \( n \) is the number of nodes.
- \( N = \{0,1,2,\ldots,n-1\} \)
- \( W_{ij} \) is the amount of flow between the origin \( i \) and destination \( j \).
- \( x \) is the collection cost (from origin spoke to hub).
- \( \beta \) is the distribution cost (from hub to destination spoke).
- \( d_{ik} \) represents the distance between nodes \( i \) and hub \( k \).
- \( d_{ij} \) is the distance between hub \( l \) and node \( j \).
- \( X_{ijk} \) is the decision variable that represents the fraction of traffic between origin node \( i \) to destination node \( j \) through hubs \( k \) and \( l \).
- \( F_{i} \) is the cost of establishing node \( i \) as hub.
- \( Z_{ij} \) is 1 if node \( i \) is assigned to hub \( j \), otherwise it is 0.
- \( Z_{ik} \) is 1 if node \( k \) is also a hub, otherwise it is 0.

Constraint (2) ensures that all the traffic between an origin-destination pair has been routed via the hub sub-network. Constraint (3) prevents non-hub nodes from being allocated to other non-hub nodes while Constraint (4) restricts the commodity flow through each hub. For some hub-spoke networks e.g., a mail delivery system, the problem may not be symmetric i.e., \( W_{ij} \neq W_{ji} \). Additionally, it may be the case that \( W_{ij} > 0 \) so that a node may route commodities to itself. In this model, both symmetric and non-symmetric flows are employed.

In another mathematical formulation, Contreras and et al [27] proposed a model for Single Allocation Hub Location Problem. This model consists of selecting a set of hubs to be established and an allocation pattern that fully assigns each node to one of the chosen hubs, that does not violate the capacity constraint of the hubs, of minimum total cost.

\[
\begin{align*}
\min Z &= \sum_{k=1}^{N} f_{k} z_{kk} + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{m=1}^{N} F_{ijkm} X_{ijkm} \quad (7)
\end{align*}
\]
Constraint (8) ensures that for each pair of nodes there is one single path connecting them, whereas constraint (9) imposes that no customer is assigned to a node that is not a hub. Constraint (10) states that if node i is assigned to hub k then all the flow from node i to any other node j must go through some other hub m. Constraint (11) has a similar interpretation relative to the flow arriving to a node j assigned to hub m from some node i. Note that constraint (8) together with constraints (10) and (11) ensure that every node is assigned to one single hub. In addition, given that the z variables are binary, they guarantee the integrality of the x variables. Constraint (12) ensures that the overall incoming flow of nodes assigned to a hub does not exceed its capacity. Constraint (13) is the aggregated demand constraint. This constraint is redundant in model, since it can be derived by adding up all constraints (12), and taking into account equalities (8) and (10).

Model similar to this model but without the aggregated demand constraint (13) was proposed by Campbell [14], and was computationally tested by Ernst and Krishnamoorthy in [12]. Computational experiments showed that the formulation that used variables with four indices lead to tighter LP bounds than the one obtained with a formulation that used variables with three indices, at a considerable increase on the required CPU times. Contreras and et al proposed a Lagrangean Relaxation associated with constraints (10) and (11). In particular, weighting constraints (10) and (11) in a Lagrangean fashion, with multipliers vectors u and v of appropriate dimensions, they obtain a Lagrangean function which, after some algebra can be expressed as \( L(u, v) = L_z(u, v) + L_x(u, v) \). The following two propositions were proven in [27]. But we proposed a model with two indices to solve Single Allocation Hub Location Problems faster than model with four indices. Additionally, for example in UCSAHLP with 10 nodes, the proposed model will have 67 constraints but Naeem’s model and Contreras’s model will have 220 and 330 constraints. Therefore, the proposed model will able to solve problems faster than models of Naeem and Contreras. We present the data, decision variables and model for computing hub location in a network.

**Research Methodology**

Hub-and-spoke problem is NP-hard problem that frequently applies in the design of transportation and distribution systems. This paper has investigated some models with un-capacitated SAHLP that have been modeled by four indices. Many indices cause to get large model and to get sluggish model solving. Thus, we proposed a model with two indices to solve Single Allocation Hub Location Problems faster than model with four indices. Additionally, for example in UCSAHLP with 10 nodes, the proposed model will have 67 constraints but Naeem’s model and Contreras’s model will have 220 and 330 constraints. Therefore, the proposed model will able to solve problems faster than models of Naeem and Contreras. In the next section, following presented proposal model.

**Data**

- \( n \) is the number of nodes.
- \( x_{ij} \) is the distribution cost (from hub to destination spoke).
- \( d_{ij} \) represents the distance between nodes i and j.
- \( F_i \) is the cost of establishing node i as hub.
- \( C_i \) is the capacity of hub i.

**Decision variables**

- \( w_{ij} \) is the amount of flow between the hub i and destination j.
- \( Z_{ij} \) is 1 if node j is assigned to hub i, otherwise it is 0.
- \( Z_{ii} \) is 1 if node i is also a hub, otherwise it is 0.
- \( h \) is the number of hubs.
Un-capacitated Model

\[
\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ x_{ij} (w_{ij} Z_{ij}) \right] + \sum_{i=1}^{n} f_{i} Z_{ii} \tag{16}
\]

\[
\sum_{i=1}^{n} Z_{ii} = h \tag{17}
\]

\[
\sum_{i=1}^{n} \sum_{j=i+1}^{n} Z_{ij} = n - h \tag{18}
\]

\[
Z_{ii} + Z_{ij} = 2 \left[ Z_{ij} + Z_{ji} \right] \quad \forall \; i, j = \{i + 1, \ldots, n\} \tag{19}
\]

\[
Z_{ii} \sum_{j=1}^{n} Z_{ij} \geq 2Z_{ii} \quad \forall \; i \tag{20}
\]

\[
(1 - Z_{ii}) \sum_{j=1}^{n} Z_{ij} \leq 1 \quad \forall \; i \tag{21}
\]

\[
w_{ij} \geq 1 \quad \forall \; i, j \tag{22}
\]

In this model, the goal of objective function (16) is to minimize the total cost of distribution and establishment. Constraint (17) counts number of hubs in optimized model. This constraint limits space of feasible solutions and model searches rapidly. Constraint (18) ensures that all the nodes have been used in model. Constraints (17) and (18) consider h nodes for hubs and n-h nodes for spokes. Constraint (19) determines to be similar routes of traffic between two nodes. Constraints (20) and (21) ensure that if a node is hub, it can connect to one or more nodes otherwise connect to one node only. Finally constraint (22) restricts lower bound of commodity flow through each node. This set of constraints ensures that all of the conditions of hub and spoke network are considered.

Results

In traditional models, nodes of origin and destination have been separated from hub and spoke nodes. But in this model, we have considered these nodes together. Each origin has considered hub and traffic between two nodes is similar. Benefit of this model is two indices that make less constraint. We now report on the evaluation of the effectiveness of the proposed model with less indices and constraints in solving the hub and spoke problems. For this report, we compared time of solving for three discussed models.

For example, in sample 01, we have chosen 9 nodes and modeled problem for it. After solving this problem with 9 nodes, model has proposed 2 optimal nodes for hub and 7 nodes for spoke with optimal cost 3650. We have solved this problem 3 times, two times by Contreras’s model and Naeem’s model in 20 seconds and one time by proposed model in 15 seconds. Less time of solving by proposed model is sake of less indices and constraints.

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Study on Marine Movements in Eastern Canada

Under a hub-and-spoke transport arrangement, parcels, freight, and/or persons are transported to a central “hub” facility, then onward to interacting nodes via a network of “spokes”. The shipping industry has also seen the emergence of massive hub ports at a variety of locales around the world, serving smaller regional “feeder” ports, particularly in the context of container shipping. The hub-and-spoke transport concept is predicated on transhipment of commodities (from one marine vessel to another) and “feeder” of commodities via marine transport to other regional ports. Figure 2 provides an overview of the marine hub-and-spoke concept [30].

Here are several examples of successful international hub-and-spoke feeder services, including the “classic” hub-and-spoke network, such as the Port of Hamburg, which serves as a hub for traffic destined to the Baltic (as well as a gateway to mainland Europe); and the “pure” transhipment hub, such as Gioia Tauro in Italy, which has a transhipment incidence of over 95 percent with little or no gateway business. These and other international examples of hub-and-spoke networks and related services are profiled in the paper and supporting working papers.

There have been few successful hub-and-spoke operations in eastern Canada, for a variety of reasons that are addressed in the paper. An example of a regional hub is the Port of Halifax. Various services have operated feeder service using Halifax as a hub (with varying degrees of success). Drawing on international examples, as well as lessons from the Canadian experience, a number of key success factors for the development of hub-and-spoke networks and related feeder services were identified:

- A critical mass of feeder traffic from/to a hub (consistency and reliability of volumes)
- Reliable, year-round access to feeder routes that serve key markets
- Competitive advantage of sea routes relative to alternative rail and road routes
- Low transhipment and handling fees at hub and feeder ports
- A regulatory environment that is conducive to investment in marine transport

Ultimately, the success of a hub-and-spoke network is contingent upon the commercial viability of the individual feeder services operating between hub and end markets. In some cases, where feeder service start-up risks are high or cost-prohibitive, support programs, such as the European Marco Polo Program or others, such as recent investment support for short sea-related infrastructure in British Columbia, can act as a catalyst to promote the development of new feeder services. For example by solving of hub-and-spoke network model has been determined routes between hubs and spokes, in figure 3 presented all short sea and ferry services in Eastern Canada as of September 2008.
Eastern Canada’s experience with hub-and-spoke feeder operations has largely been successful. The advantages of hub-and-spoke and regional short sea operations include the following [30]:

- Lower transport costs per tonne/kilometre than road transport
- Additional (or better utilization of) transport capacity, particularly where competing road transport experiences capacity constraints
- Feeders offer wider market coverage for a gateway
- Feeders can offer container service to markets not big enough to be served by direct call
- Less long-haul trucking required (in the case of regional short sea services), and related wear and tear on roads
- Lower environmental impacts and social costs

Conclusions

In this paper we have presented a model with less indices for the Un-Capacitated Single Allocation Hub Location Problems. Comparison results of three models (Naeem, Contreras and Proposed model) represented that the proposed model was faster than others. In addition, Naeem's model was faster than Contreras's model due to less constraint. We proposed the model to solve instances of SAHLP up to 50 nodes. To the best of our knowledge, the considered instances are the small and medium instances that have been solved exactly for the UCSAHLP in an acceptable time.

References